American Journal of Sciences and Engineering Research
E-ISSN -2348 – 703X, Volume 4, Issue 2, 2021

Pure Bending Analysis of Thin Rectangular Flat Plate with All Edges Simply Supported Carrying Uniformly Distributed Load Using Euler-Bernoulli Residual Force Approach

Anyaogu L.[1], Ibearugbulem O. M.[2], Christopher G. O.[3*], Azie G. O.[4]

[1, 2, 3] *Department of Civil Engineering, Federal University of Technology, P.M.B 1526, Owerri, Imo State, Nigeria*

[4] *Department of Civil Engineering, Imo State University, P.M.B 2000, Owerri, Imo State, Nigeria*

ABSTRACT: *This study investigates pure bending analysis of thin rectangular flat plate with all edges simply supported carrying uniformly distributed load using Euler-Bernoulli residual force equilibrium equation. The analysis was accomplished by carrying out direct differentiation of total potential energy functional of a rectangular flat plate with respect to the displacement function, w (x, y) to obtain the general Euler-Bernoulli residual force equilibrium equation for the plate. The study used direct integration to solve the Euler-Bernoulli residual force equilibrium equation of plates to obtain the exact general deflection equation with unknown coefficients. The boundary conditions of the all edges simply supported (SSSS) plate were satisfied to obtain the particular solution for SSSS plate. Euler-Bernoulli residual force equilibrium equation was at this point used to obtain the exact coefficient of deflection, A by employing the particular solution of SSSS plate obtained earlier on. With the exact shape functions and the corresponding exact coefficient of SSSS plate obtained, the study went on to determine the exact central deflection and exact maximum bending moments for the SSSS plate. The result of the present study was compared with the results of Ibearugbulem (2014). The results obtained herein showed an average percentage difference of 23.47% between the present study and Ibearugbulem, 2014. The method is simple and devoid of complexity.*

KEYWORDS: *Euler-Bernoulli Residual Force, Weighted Residual Force, Shape Function, Coefficient of Deflection, Partial Differential Equation.*

NOTATIONS

A: Coefficient of deflection, a and b: Rectangular plate lateral dimensions, βx: Coefficient of maximum moment in x direction, βy: Coefficient of maximum moment in y direction, S: Simple support, SSSS: Four edges are simply supported, D: Modulus of flexural rigidity of the plate, E: Young's modulus, ε: Normal strain, F: Euler-Bernoulli form of total equilibrium of forces, $\bar{F}$: Weighted residual form of total equilibrium of forces, G: Torsional modulus of elasticity of the plate, g: Euler-Bernoulli form of equilibrium of forces at an arbitrary point, h: Shape function of the plate under consideration, K_D: Coefficient of maximum deflection, Ksx: Coefficient of maximum shear force in x direction, Ksy: Coefficient of maximum shear force in y direction, M_x: Moment in x direction, M_y: Moment in y direction, M_{xy}: Moment in x-y direction, M_x: Moment in x direction, P: Aspect ratio of rectangular plate. That is P = a/b, Q: Non dimensional axis (quantity) parallel to y axis. Q = y/b, q: Distributed load intensity, R: Non dimensional axis (quantity) parallel to x axis, R = x/a, t: Plate thickness, U:

 Received- 24-12-2020, Accepted- 04-02-2021

Internal (strain) energy, X: The primary axis of the plate. That is the shorter of the two axes of the major plane of the plate, Y: The secondary axis of the plate. That is the longer of the two axes of the major plane of the plate, Z: The tertiary axis of plate. That is the shortest of the three axes of the plate, W = w (x, y): Plate displacement in z direction. It is a function of x and y, Π: Potential energy functional of the plate, μ: Poisson's ratio, σ: Normal stress of the plate, τ: Shear stress of the plate, γ: Shear strain of the plate

I. INTRODUCTION

A flat plate, like a straight beam carries lateral load by bending, Ibearugbulem (2014). Plate bending analyses is categorized into two types based on thickness to breadth ratio: Thick plate and Thin plate analyses. According to Timoshenko & Woinosky-Krieger (1959), if the thickness to width ratio of the plate is less than 0.1 and the maximum deflection is less than one tenth of thickness, then the plate is classified as thin plate. The well-known Kirchhoff plate theory is used for the analysis of such thin plates. On the other hand, Mindlin plate theory is used for thick plate where the effect of shear deformation is included, Mindlin (1951). The Kirchhoff–Love theory is an extension of Euler–Bernoulli beam theory to thin plates and was developed in 1888 by Love, A. E. H. (1888). The classical beam theory was first applied to plates and shells by Love and Kirchhoff, (Reddy, 2007). Kirchhoff–Love plate theory is commonly known as Kirchhoff's plate theory.

The general Euler-Bernoulli theory for a continuum in equilibrium can be represented mathematically as given in Equation (1.1);

$$F = \frac{d\Pi}{dw} = \iint \left[\Delta w - \frac{q}{D}\right] dx\, dy = 0 \qquad (1.1)$$

Where;

w = displacement function,

Δw = derivative of the displacement function

Π = total potential energy functional of a plate

D = flexural rigidity of the plate,

dx = elemental length in x − axis,

dy = elemental distance in y − axis and

q is the applied load on the plate.

The displacement function, w in Equation (1.1) is substituted with a deflection function, h which is often preselected such that the specified boundary conditions of the problem are satisfied. The preselected deflection (shape) function, h has unknown coefficient, A. substituting the shape function, h and the coefficient, A, for the displacement function, w, in Equation (1.1) results to Equation (1.2).

$$F = \frac{d\Pi}{dw} = \iint \left[A\Delta h - \frac{q}{D}\right] dx\, dy = 0 \qquad (1.2)$$

A = coefficient of deflection,

Δh is derivative of deflecion(shape) function

Rearrangement of Equation (1.2), gave rise to Equation (1.3);

$$A = \frac{q/D}{\iint \Delta h dx dy} \qquad (1.3)$$

The coefficient of the deflection function, A is obtained from Equation (1.3) such that Equation (1.1) is satisfied in the exact sense. Therefore, the residual force from Equation (1.1) is zero. This agrees with Newton's third law of motion which states that 'For every action, there is an equal and opposite reaction'. This is the sole condition for static equilibrium. Any analysis that circumvents Equations (1.1) and (1.3) in its process will result in approximate solution.

Ibearugbulem, (2013) noted that earlier scholars such as Navier (1823) and Levy (1899) assumed some functions in form of trigonometric series that satisfied the boundary conditions of the particular plates and substituted them into the governing equation (Equation (1.1)) before integration. One of the notable limitations to this approach is the difficulty in assuming a satisfying shape function. This led to the adoption of the weighted residual method by later scholars. In weighted residual method (WRM), an approximate solution of a boundary value problem is obtained by using the corresponding integral formulation. An approximate form for the solution (deflection function, h) is assumed in terms of a series containing known functions and unknown coefficients (Reddy, 2007). When this form is substituted in the integral formulation, a set of algebraic equations in terms of the unknown coefficients was obtained. Solution of the algebraic equations determines the coefficients.

Multiplying Equation (1.1) with any weighting function for instance the shape function h, yielded Equation (1.4).

$$\mathrm{Fh} = \frac{d\Pi}{dw}\mathrm{h} = \iint \left[\Delta w.\mathrm{h} - \frac{q}{D}\right] \mathrm{h}\,\mathrm{dxdy}$$
$$= \iint \left[A\Delta h.\mathrm{h} - \frac{q}{D}\mathrm{h}\right] \mathrm{dxdy} = 0 \qquad (1.4)$$

Equation (1.4) is the weighted residual force equation of a plate. It is similar to Equation (1.1). Equation (1.1) is the true force equation, while Equation (1.4) is the quasi-force equation. Like Equation (1.3), the coefficient of the deflection function, A, can similarly be calculated by making A, the subject of the formula in Equation (1.4) as given in Equation (1.5).

$$A = q/D \frac{\iint \mathrm{hdxdy}}{\iint \Delta h.\mathrm{hdxdy}} \qquad (1.5)$$

Equation (1.5) is characterized by its accelerated convergence. Hence, the sole reason why it is always employed in analysis, especially when dealing with approximate deflection functions with unknown coefficients. If the displacement function is approximate, it cannot be used in Euler-Bernoulli coefficient Equation (1.3). Most weighted residual force method (WRM) like Galerkin and Ritz employed in plate analysis assumed shape function whose boundary conditions satisfied Equation (1.4) and went on to use Equation (1.5) to determine the unknown coefficients of the shape function. The present work tried to use the residual equation (Equation (1.1)) and the coefficient equation (Equation (1.3)) to determine the maximum deflection and moment in SSSS plate type.

II. THEORETICAL BACKGROUND

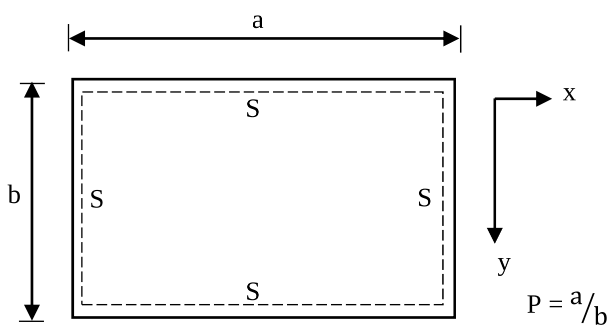

Figure 1. Schematic representation of SSSS plate carrying uniformly distributed load

Equation (2.1) is the total or overall potential energy functional for isotropic rectangular flat plate. The equation was derived by Ibearugbulem (2014) using Ritz approach.

$$\Pi = \frac{D}{2}\int_0^a \int_0^b \left(\left[\frac{\partial^2 w}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 w}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 w}{\partial y^2}\right]^2\right) dx\, dy - q\int_0^a \int_0^b w\, \partial x\, \partial y \qquad (2.1)$$

Equation (2.1) can be written in non-dimensional form in terms of the Cartesian coordinates x and y, and the lateral dimensions a and b, as shown in Equation (2.2).

$$\Pi = \frac{abD}{2a^4}\int_0^a \int_0^b \left(\left[\frac{\partial^2 w}{\partial R^2}\right]^2 + \frac{2}{p^2}\left[\frac{\partial^2 w}{\partial R\, \partial Q}\right]^2 + \left[\frac{\partial^2 w}{\partial Q^2}\right]^2\right) dR dQ - abq\int_0^a \int_0^b w\, \partial R\, \partial Q \qquad (2.2)$$

Where $R = \frac{x}{a}$, $Q = \frac{y}{b}$ and

$P = \frac{a}{b}$, the aspect ratio of the plate

$$D = \frac{Et^3}{12(1-\mu^2)} \qquad (2.3)$$

D is the flexural rigidity. E is Young's modulus of elasticity, t is the thickness of the plate, μ is the Poisson ratio and, w is the shape function (deflection function).

Equation (2.4) is obtained by expansion and further simplification of Equation (2.2).

$$\Pi = \frac{abD}{2a^4}\int_0^a \int_0^b \left(\frac{\partial^4 w}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 w}{\partial R^2\, \partial Q^2} + \frac{1}{p^4}\frac{\partial^4 w}{\partial Q^4}\right) dR\, dQ - abq\int_0^a \int_0^b dR\, dQ \qquad (2.4)$$

SOLUTION OF TOTAL ENERGY FUNCTIONAL BY EULER-BERNOULLI RESIDUAL FORCE APPROACH.

From elementary physics, Energy is defined as the product of force, F and distance, w (x). Mathematically, this is as expressed in Equation (2.5).

$$\text{Energy}, \Pi = \text{Force}, F \times \text{Distance}, w(x) \qquad (2.5)$$

Making force, F in Equation (2.5) the subject of the relationship yields Equation (2.6).

$$\text{Therefore, Force}, F = \frac{d\text{Energy}}{dw(x)} = \frac{d\Pi}{dw} \qquad (2.6)$$

Differentiating Equation (2.6) with respect to deflection, w gives resultant force as zero.

$$F = \frac{d\Pi}{dw} = \int_0^1 \int_0^1 \left(\frac{\partial^4 w}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 w}{\partial R^2\, \partial Q^2} + \frac{1}{p^4}\frac{\partial^4 w}{\partial Q^4} - \frac{qa^4}{D}\right) dR\, dQ = 0 \qquad (2.7)$$

Let deflection be defined as:

$$w = Ah \qquad (2.8)$$

Where A is Coefficient of deflection and h is shape function (deflection function)

Substituting Equation (2.8) into Equation (2.7) gives and simplifying yields Equation (2.9):

$$A\int_0^1 \int_0^1 \left(\frac{\partial^4 h}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 h}{\partial R^2\, \partial Q^2} + \frac{1}{p^4}\frac{\partial^4 h}{\partial Q^4}\right) dR\, dQ = \frac{qa^4}{D} \qquad (2.9)$$

That is:

$$A[k_1 + k_2 + k_3] = \frac{qa^4}{D} \qquad (2.10)$$

Where:

$$k_1 = \int_0^1 \int_0^1 \frac{\partial^4 h}{\partial R^4} dR\, dQ \qquad (2.11)$$

$$k_2 = \int_0^1 \int_0^1 \frac{2}{p^2}\frac{\partial^4 h}{\partial R^2\, \partial Q^2} dR\, dQ \qquad (2.12)$$

$$k_3 = \int_0^1 \int_0^1 \frac{1}{p^4}\frac{\partial^4 h}{\partial Q^4} dR\, dQ \qquad (2.13)$$

Thus, rearranging Equation (2.10) gives:

$$A = \frac{1}{[k_1 + k_2 + k_3]}\frac{qa^4}{D} \qquad (2.14)$$

Let Equation (2.14) be represented as shown in Equation (2.15).

$$A = k_T \frac{qa^4}{D} \qquad (2.15)$$

Where:

$$k_T = \frac{1}{[k_1 + k_2 + k_3]} \qquad (2.16)$$

Substituting Equation (2.15) into Equation (2.10) yields Equation (2.17).

$$k_T \frac{qa^4}{D}\ [k_1 + k_2 + k_3] = \frac{qa^4}{D} \qquad (2.17)$$

On further simplification of Equation (2.17), Equation (2.18) is obtained.

$$k_T\ [k_1 + k_2 + k_3] = 1 \qquad (2.18)$$

Or

$$k_T\ k_1 + k_T k_2 + k_T k_3 = 1 \qquad (2.19)$$

]For the integral of Equation (2.7) to be zero, it implies that the integrand is zero. This is expressed in Equation (2.20)

$$\frac{\partial^4 w}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} + \frac{1}{p^4}\frac{\partial^4 w}{\partial Q^4} - \frac{qa^4}{D} = 0 \qquad (2.20)$$

Substituting Equation (2.19) into Equation (2.7) yields Equation (2.21).

$$\int_0^1\int_0^1 \left(\frac{\partial^4 w}{\partial R^4} + \frac{2}{p^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} + \frac{1}{p^4}\frac{\partial^4 w}{\partial Q^4} - \frac{qa^4}{D}[k_T\ k_1 + k_T k_2 + k_T k_3]\right) dRdQ = 0 \qquad (2.21)$$

This can be rearranged as shown in Equation (2.22).

$$\int_0^1\int_0^1 \left(\left[\frac{\partial^4 w}{\partial R^4} - \frac{qa^4}{D}k_T\ k_1\right] + \left[\frac{2}{p^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} - \frac{qa^4}{D}k_T k_2\right] + \left[\frac{1}{p^4}\frac{\partial^4 w}{\partial Q^4} - \frac{qa^4}{D}k_T k_3\right]\right) dR\ dQ = 0 \qquad (2.22)$$

One of the conditions for which Equation (2.22) will be true is if each of these integrals is zero. That is:

$$\int_0^1\int_0^1 \left(\left[\frac{\partial^4 w}{\partial R^4} - \frac{qa^4}{D}k_T\ k_1\right]\right) dR\ dQ = 0 \qquad (2.23)$$

$$\int_0^1\int_0^1 \left(\left[\frac{2}{p^2}\frac{\partial^4 w}{\partial R^2\,\partial Q^2} - \frac{qa^4}{D}k_T k_2\right]\right) dR\ dQ = 0 \qquad (2.24)$$

$$\int_0^1\int_0^1 \left(\left[\frac{1}{p^4}\frac{\partial^4 w}{\partial Q^4} - \frac{qa^4}{D}k_T k_3\right]\right) dR\ dQ = 0 \qquad (2.25)$$

Let us at this point split deflection into w_x and w_y as:

$$w = w_x . w_y \qquad (2.26)$$

Substituting Equation (2.26) into Equations (2.23), (2.24) and (2.25) and rearranging the resulting equations gives respectively:

$$\int_0^1\int_0^1 \left(\left[w_y \frac{\partial^4 w_x}{\partial R^4} - \frac{qa^4}{D}k_T\ k_1\right]\right) dR\ dQ = 0 \qquad (2.27)$$

$$\int_0^1\int_0^1 \left(\left[\frac{2}{p^2}\frac{\partial^2 w_x}{\partial R^2}\frac{\partial^2 w_y}{\partial Q^2} - \frac{qa^4}{D}k_T k_2\right]\right) dR\ dQ = 0 \qquad (2.28)$$

$$\int_0^1\int_0^1 \left(\left[\frac{w_x}{p^4}\frac{\partial^4 w_y}{\partial Q^4} - \frac{qa^4}{D}k_T k_3\right]\right) dR\ dQ = 0 \qquad (2.29)$$

Carrying out the integration of Equations (2.27) with respect to Q and Equation (2.29) with respect to R gives respectively:

$$\int_0^1 \left(\left[w_x \frac{\partial^4 w_x}{\partial R^4} - \frac{qa^4}{D} k_T\, k_1 \right] \right) dR \; = 0 \qquad (2.30)$$

$$\int_0^1 \left(\left[\frac{w_y}{p^4} \frac{\partial^4 w_y}{\partial Q^4} - \frac{qa^4}{D} k_T k_3 \right] \right) \, dQ = 0 \qquad (2.31)$$

For Equations (2.30) and (2.31) to be true, their integrands must be zero. That is:

$$\frac{\partial^4 w_x}{\partial R^4} = \frac{qa^4}{Dw_x} k_T\, k_1 \qquad (2.32)$$

$$\frac{\partial^4 w_y}{\partial Q^4} = \frac{qa^4}{Dw_y} k_T k_3 p^4 \qquad (2.33)$$

Let:

$$A_x = \frac{qa^4}{Dw_x} k_T\, k_1 \qquad (2.34)$$

$$A_y = \frac{qa^4}{Dw_y} k_T k_3 p^4 \qquad (2.35)$$

$$A = \frac{A_x}{24} . \frac{A_y}{24} \qquad (2.36)$$

Equating Equations (2.32) and (2.34), and (2.33) and (2.35), shows that;

$$\frac{\partial^4 w_x}{\partial R^4} = A_x \qquad (2.37)$$

$$\frac{\partial^4 w_y}{\partial Q^4} = A_y \qquad (2.38)$$

Solving Equations (2.37) and (2.38) by direct integration shall respectively give:

$$w_x = a_0 + a_1 R + a_2 \frac{R^2}{2} + a_3 \frac{R^3}{6} + A_x \frac{R^4}{24} \qquad (2.39)$$

$$w_y = b_0 + b_1 Q + b_2 \frac{Q^2}{2} + b_3 \frac{Q^3}{6} + A_y \frac{Q^4}{24} \qquad (2.40)$$

Multiplying Equations (2.39) and (2.40) gives:

$$w = w_x . w_y$$

$$= \left(a_0 + a_1 R + a_2 \frac{R^2}{2} + a_3 \frac{R^3}{6} + A_x \frac{R^4}{24} \right) . \left(b_0 + b_1 Q + b_2 \frac{Q^2}{2} + b_3 \frac{Q^3}{6} + A_y \frac{Q^4}{24} \right) \qquad (2.41)$$

Particular solution for SSSS plate

The boundary conditions of SSSS plate

$$w(R = 0) = 0;\; w''^{R}(R = 0) = 0 \qquad (2.42)$$

$$w(R = 1) = 0;\; w''^{R}(R = 1) = 0 \qquad (2.43)$$

$$w(Q = 0) = 0;\; w''^{Q}(Q = 0) = 0 \qquad (2.44)$$

$$w(Q = 1) = 0;\; w''^{Q}(Q = 1) = 0 \qquad (2.45)$$

Substituting these boundary conditions into Equation (2.41) and solving gives;

$$a_0 = 0, a_1 = \frac{A_x}{24}, a_2 = 0, a_3 = -\frac{A_x}{2}, \text{and}$$

$$b_0 = 0, b_1 = \frac{A_x}{24}, b_2 = 0, b_3 = -\frac{A_y}{2}$$

Substituting back these constants into Equation (2.41) yields Equations (2.46).

$$w = A(R - 2R^3 + R^4).(Q - 2Q^3 + Q^4) \qquad (2.46)$$

The exact deflection function, h for a SSSS plate is given as in Equation (2.47)

$$h = (R - 2R^3 + R^4).(Q - 2Q^3 + Q^4) \quad (2.47)$$

Determination of the exact coefficient of deflection of a SSSS plate.

Partial differentiation and integration of Equation (2.46) gave the following stiffness coefficient values for SSSS plate. The effective length of a SSSS plate is 1.0L. The lower limit is 0 and the upper limit is 1. Hence, the limits of this integration are 0 and 1.

$$k_x = \int_0^1 \int_0^1 \frac{d^4w}{dR^4} dRdQ = 4.8 \quad (2.48)$$

$$k_{xy} = \int_0^1 \int_0^1 \frac{d^4w}{dR^2dQ^2} dRdQ = 4.0 \quad (2.49)$$

$$k_y = \int_0^1 \int_0^1 \frac{d^4w}{dQ^4} dRdQ = 4.8 \quad (2.50)$$

$$k_q = \int_0^1 \int_0^1 dRdQ = 1.0 \quad (2.51)$$

Making A, the subject of Equation (2.9) and simplifying the resulting equation will yield the general equation for exact coefficient of deflection of an isotropic flat plate subjected to uniformly distributed load as expressed in Equation (2.52).

$$A = \frac{\frac{qa^4}{D} k_q}{k_x + \frac{2}{p^2} k_{xy} + \frac{1}{p^4} k_y} \quad (2.52)$$

Substituting the values of Equation (2.48), Equation (2.49), Equation (2.50) and Equation (2.51) into Equation (2.52) yields the particular exact coefficient of deflection for SSSS plate.

$$A = \frac{qa^4}{D} \frac{1.0p^4}{(4.8p^4 + 8.0p^2 + 4.8)} \quad (2.52)$$

In non – dimensional form, for critical (midspan) deflections and moments will occur at the center of the plate where:

$$R = 0.5 \text{ and } Q = 0.5 \quad (2.53)$$

$$W_{max} = \left(\left(\frac{25}{256} \right) \frac{1.0p^4}{(4.8p^4 + 8.0p^2 + 4.8)} \right) \frac{qa^4}{D} \quad (2.54)$$

Let maximum deflection be represented as shown in Equation (2.55)

$$W_{max} = k_D \frac{qa^4}{D} \quad (2.55)$$

Where kD,

$$k_D = \left(\left(\frac{25}{256} \right) \frac{1.0p^4}{(4.8p^4 + 8.0p^2 + 4.8)} \right) \quad (2.56)$$

Mid-span moment in x and y directions.

$$M_x = -D \left[\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right] \quad (2.57)$$

Equation (2.57) is bending moment equation of a two-dimensional element from elementary theory of structure. Substituting Equation (2.3) and the second derivatives of Equation (2.46) into Equation (2.57) for Poisson ratio of 0.3, R = Q = 0.5 yields Equation (2.58).

$$M_x = \left[\frac{0.281250}{P^2} + 0.93750\right]\frac{1.0p^4}{(4.8p^4 + 8.0p^2 + 4.8)}\, qa^2 \tag{2.58}$$

Let maximum bending moment in x - direction be represented as shown in Equation (2.59)

$$M_x = \beta_x qa^2 \tag{2.59}$$

Where βx is;

$$\beta_x = \left[\frac{0.281250}{P^2} + 0.93750\right]\frac{1.0p^4}{(4.8p^4 + 8.0p^2 + 4.8)} \tag{2.60}$$

Similarly,

$$M_{yC} = \left[\frac{0.93750}{P^2} + 0.281250\right]\frac{1.0p^4}{(4.8p^4 + 8.0p^2 + 4.8)}\, qa^2 \tag{2.61}$$

Let maximum bending moment in y - direction be represented as shown in Equation (2.59)

$$M_y = \beta_y qa^2 \tag{2.62}$$

Where βy is;

$$\beta_{yC} = \left[\frac{0.93750}{P^2} + 0.281250\right]\frac{1.0p^4}{(4.8p^4 + 8.0p^2 + 4.8)} \tag{2.63}$$

III. RESULTS AND DISCUSSIONS

This study successfully obtained the exact deflection and coefficient of deflection using for an isotropic rectangular flat plate subjected to uniformly distributed load, q. the equations are reproduced here as Equations (2.64) and (2.65). The numerical values of deflection and bending moment factors at the centre of the plate for different aspect ratios are shown on the Table 1 and Table 2. Values of deflection factor obtained in the present study were compared with those obtained by Ibearugbulem (2014). Similarly, the values of bending moments obtained from this study were compared with those obtained in Ibearugbulem (2014). The average percentage difference between the two studies are 12.46% and 12.44% for deflection and bending moments in x and y directions respectively.

The shape function obtained in the present study turns out to be the same as those assumed in the weighted residual approach. The coefficient of deflection however varied from those obtained in previous studies. This is the sole reason for variation in the two results. Since the results obtained from this study satisfies not only the plate boundary conditions but also satisfies Euler-Bernoulli residual force equation, and the exact coefficient of deflection equation was also used to obtained the coefficient of deflection in the present study, It is therefore recommend that this new Euler-Bernoulli residual force approach for isotropic rectangular flat SSSS plate analysis could be more easily used over the traditional weighted residual approximate methods.

$$w = A(R - 2R^3 + R^4)\,.\,(R - 2Q^3 + Q^4) \tag{2.64}$$

$$A = \frac{qa^4}{D}\frac{1.0p^4}{(4.8p^4 + 8p^2 + 4.8)} \tag{2.65}$$

Table 1. Values of deflection, kD

P	kD from present study	kD from Ibearugbulem (2014)	% difference b/w present study & Ibearugbulem (2014)
1.0	0.00555	0.00414	25.39
1.1	0.00665	0.00496	25.39
1.2	0.00771	0.00576	25.27
1.3	0.00871	0.00653	25.01
1.4	0.00964	0.00725	24.79
1.5	0.01050	0.00793	24.45
1.6	0.01128	0.00856	24.11
1.7	0.01199	0.00913	23.87
1.8	0.01264	0.00966	23.57
1.9	0.01322	0.01015	23.25
2.0	0.01375	0.01059	23.01

Table 2. Values of maximum bending moment in x and y directions

P	βx, present study	βx, Ibearugbulem (2014)	% diff. b/w present study& Ibearugbulem (2014)	βy, present study	βy, Ibearugbulem (2014)	% diff. b/w previous study & present study
1.0	0.06925	0.05163	25.44	0.06925	0.05163	25.44
1.1	0.07964	0.05943	25.38	0.07189	0.05364	25.38
1.2	0.08941	0.06685	25.23	0.07358	0.05502	25.22
1.3	0.09844	0.07383	25.00	0.07455	0.05591	25.00
1.4	0.10670	0.08031	24.73	0.07497	0.05643	24.73
1.5	0.11420	0.08628	24.45	0.07501	0.05668	24.44
1.6	0.12098	0.09176	24.15	0.07479	0.05673	24.14
1.7	0.12708	0.09677	23.85	0.07438	0.05664	23.85
1.8	0.13257	0.10133	23.57	0.07385	0.05645	23.56
1.9	0.13751	0.10549	23.28	0.07326	0.05620	23.28
2.0	0.14195	0.10927	23.02	0.07262	0.05591	23.01

IV. REFERENCES

[1] A. E. H. Love, *On the small free vibrations and deformations of elastic shells*, Philosophical trans. of the Royal Society (London), 1888, Vol. Série A, N° 17 p. 491–549.

[2] Ancel, C. U., Soul, K. F. (2003). *Advanced Strength and Applied Elasticity.* New Jersey: Prentice Hall.

[3] Finlayson, B. A. (1972). *The Method of Weighted Residuals and Variational Principles.* New York: Academic Press.

[4] J. C. Ezeh, L.O. Ettu, O. M. Ibearugbulem. (2013). *Direct Integration and Work Principle as New Approach in Bending Analyses of Isotropic Rectangular Plates*. The International Journal of Engineering And Science (IJES), 28-36.

[5] Ibearugbulem, O. M, Anyaogu, L, Christopher, G. O. (2020). *Pure Bending Analysis of Thin Rectangular Flat Plate with all Edges Clamped Carrying Uniformly Distributed Load Using Euler-Bernoulli Residual Force Approach*. The International Journal of Scientific & Engineering Research (IJSER), 202-208.

[6] Ibearugbulem, O.M, Ettu, L. O., Ezeh, J. C., (2014). *Energy Methods in Theory of Rectangular Plates.* Owerri: Liu House of Excellence Ventures.

[7] R. D. Mindlin, *Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates*, Journal of Applied Mechanics, 1951, Vol. 18 p. 31–38.

[8] Reddy, J. N. (1984). *Energy and Variational Methods in Applied Mechanics.* Texas: John Willey & Sons.

[9] Reddy, J. N. (1993). *Introduction to the Finite Element Method.* USA: McGraw-Hill, Inc.

[10] Reddy, J. N. (2007). *Theory and analysis of elastic plates and shells.* CRC Press.
[11] Szilard Rudolph (2004). *Theories and Applications of Plates Analysis. Classical, Numerical and Engineering methods*. John Wiley and Sons Inc. Hoboken USA.
[12] Timoshenko, S. P. and Goodier, J. N. (1970). *Theory of Elasticity.* New York: McGraw-Hill.
[13] Timoshenko, S., Woinosky-Krieger, S. (1959). *Theory of plates and shells.* New York: McGraw-Hill.
[14] Ugural, A. C. and Fenster, S. K. (1975). *Advanced Strength and Applied Elasticity.* New York: Elsevier.
[15] Ventsel, E., & Krauthammer, T. (2001). *Thin Plates and Shells: Theory, Analysis and Applications.* New York: Maxwell Publishers Inc.
[16] Washizu, K. (1975). *Variational Methods in Elasticity and Plasticity, 2nd ed.* New York: Pergamon Press.
[17] Weinstock, R. (1952). *Calculus of Variations with Applications to Physics and Engineering.* New York: McGraw-Hill.