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Numerical Simulation of Sediment Transport in Shallow Water

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ABSTRACT: *A scientific challenge is the study of the evolution of the morphodynamics of the odds. Indeed, it should be noted that more than half of the world's population lives in coastal areas susceptible to erosion. Many approaches have been used to better understand the process of sediment transport and deposition, including prediction, stochastic modeling, mathematical models, and physical models. In this paper we present a numerical model that allows the simulation of the morphologic behavior of seabed. Sediment transport in the seabed is presented by a system of one-dimensional equations describing a water-sediment mixture and translating the laws of conservation. These equations form the Saint-Venant-Exner model. The numerical method used is a finite element method implanted in Multiphysics COMSOL. The obtained results show the contribution of bed load transport and the importance of the interaction between water and sediment.*

Key words: *Seabed profile, Saint – Venant – Exner model, numerical simulation, COMSOL Multiphysics, Finite element method*

I. Introduction

Significant amounts of sand are transported in the littoral zone by the action of swells and currents. When waves hit the coast and crash onto the beach, they cause coastal currents to interact with the incident waves, causing sediment to be transported. Thus, it is essential to predict the evolution of sediment transit, particularly the detection of erosion and deposition zones at shore level. In this paper, we present a simulation of the behavior of seabed in the coastal area using a numerical approach of PDE system Saint – Venant – Exner. Our approach consists in solving the system by finite element method in COMSOL Multiphysics. To appreciate the behavior of our model, we simulate the action of the wave propagation on the mobile floor topography and we compare our results with the literatures. The rest of this article is organized as follows. In the second section, we describe the model of sediment transport by bedload in the shallow water and COMSOL Setting. In the third section, we present results and discussions on time dependent evolution of seabed and the importance of interaction parameter.

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II. Material and methods

1.1. Shallow water model

The majority of free surface flows occur under conditions where the lateral dimensions are significant in relation to the depth. [9]. Thus, the change in the value of the variables remains very small along the vertical. This finding suggests a simplification of the three-dimensional Navier-Stokes equations by using a vertical average of the three equations in the three directions, we are talking about approximations in shallow water leading to the shallow water equations also called shallow water model. We can show that the one-dimensional shallow water hydrodynamic model is governed by the shallow water equations (See [2] and [4]):

$$\begin{cases} \dfrac{\partial h}{\partial t}+\dfrac{\partial \boldsymbol{q}}{\partial \mathrm{x}}=0 \\ \dfrac{\partial \boldsymbol{q}}{\partial t}+\dfrac{\partial}{\partial x}\left(\dfrac{\boldsymbol{q}^2}{h}+g\dfrac{h^2}{2}\right)-\dfrac{\partial}{\partial x}\left(\nu\dfrac{\partial \boldsymbol{q}}{\partial x}\right)=-gh\dfrac{\partial z_b}{\partial x}-\dfrac{\boldsymbol{\tau}}{\rho} \end{cases} \tag{2.1}$$

Where h designed the thickness of the water layer, $\boldsymbol{q}=h\boldsymbol{u}$ the discharge of water, z_b the seabed topography, g the gravity constant, ρ the density of water, ν the kinematic viscosity and $\boldsymbol{\tau}$ represent the friction stress at bottom.

1.2. Exner equation

Sediment transport is characterized mainly by two processes, suspension transport and bedload transport. The Exner equation describes the process of bedload transport, and therefore bed erosion and engulfment. It characterizes the conservation of the mass of sediments (see [1] and [8]):

$$(1-p)\frac{\partial z_b}{\partial t}+\frac{\partial \boldsymbol{q_S}}{\partial x}=0 \tag{2.2}$$

With p and q_S are respectively the parameters related to the porosity and the volumetric bedload sediment transport rate.

1.3. Closing laws

To close the system Saint – Venant – Exner we need two supplementary informations:

- **Friction law:** The term friction can be defined by the formulas of Chézy or Manning – Strickler. We can be found in the many study, a relation between this friction term and the head loss J.

$$\frac{\boldsymbol{\tau}}{\rho}=gh\boldsymbol{J} \tag{2.3}$$

Here, we are going to take Manning – Strickler formulas

$$\boldsymbol{J}=\frac{|\boldsymbol{u}|\boldsymbol{u}}{K_S^2 h^{\frac{4}{3}}} \tag{2.4}$$

With K_S is the Strickler coefficient

- **Transport law:** There are many formulas for sediment transport that can be found in the literature see for example [3]. In this work, we are using Grass's expression (See [6],[7]):

$$\boldsymbol{q_S}=A_g|\boldsymbol{u}|^{m-1}\boldsymbol{u} \tag{2.5}$$

Where A_g is the parameter related to the particle size and dynamic viscosity and m a positive integer equal to 3 in most applications.

1.4. COMSOL Multiphysics setting

COMSOL is a Multiphysics package often used to represent and store physical systems that describe phenomena [12]. In this paper, we aim at applying COMSOL discretize and to solve the system Saint – Venant – Exner via the finite element method. For that we use the General form PDE interface

$$e_a \frac{\partial^2 \boldsymbol{u}}{\partial x^2} + d_a \frac{\partial \boldsymbol{u}}{\partial x} + \nabla.\boldsymbol{\Gamma} = \boldsymbol{f} \qquad (2.6)$$

In this expression, our vector $\boldsymbol{u}$ can be seen as the dependent variable matrix transposed and denoted:

$$\boldsymbol{u} = [h, \boldsymbol{q}, z_b]^T \qquad (2.7)$$

And the conservative flux $\boldsymbol{\Gamma}$ was represented as the 3 by 1 matrix

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{q} \\ -\nu \frac{\partial \boldsymbol{q}}{\partial x} \\ \boldsymbol{q_s} \end{bmatrix} \qquad (2.8)$$

We applied the initials conditions for :

- the seabed

$$z_b(x,0) = \frac{0.08}{1 + e^{0.5(6.5 - x)}} + 0.014 e^{-(x-7.5)^2} \qquad (2.9)$$

- the sea surface

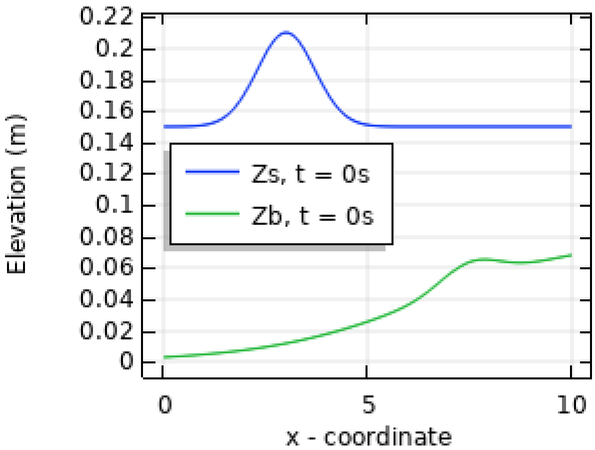
$$z_s(x,0) = 0.15 - z_b(x,0) + 0.06 e^{-(x-3)^2} \qquad (2.10)$$

III. Results et discussions

The main aim of this study is to show that evolution of seabed and the importance of the parameter interaction which depend on characteristics of seabed and the hydrodynamic of the fluid. For that we realized a 1D simulation of the sediment transport under action we wave propagation in the canal 10m length.

1.5. Numerical result of time depend evolution of seabed

The figure 1 represent the level of sea surface z_s and the seabed z_b for different time with interaction parameter value $A_g = 0.2$.



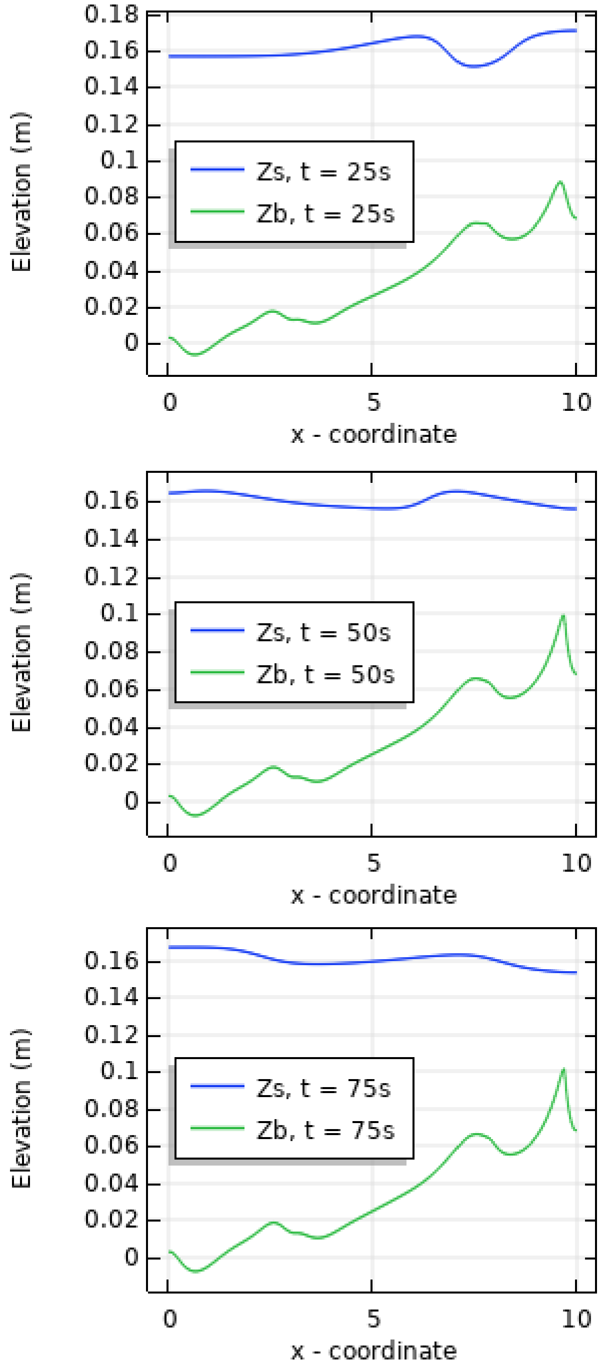
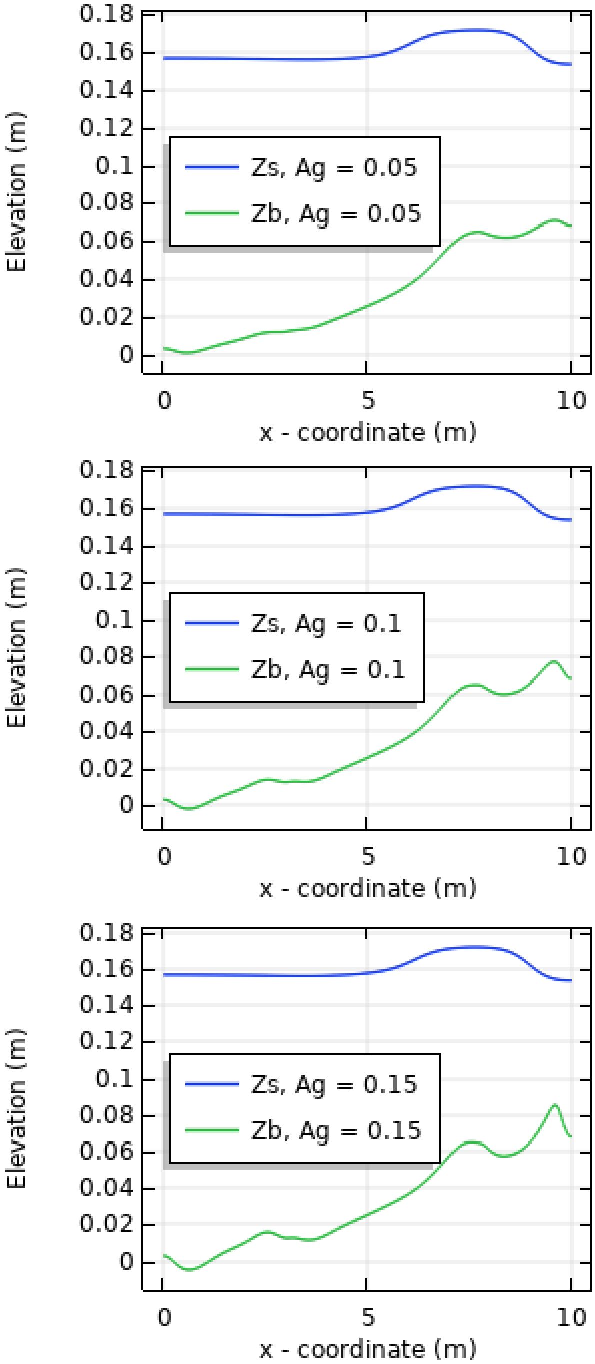


Figure 1. Evolution of sea bed under influence of the shallow water for t = 0 s, t = 25 s, t = 50 s, t = 75s

We can see on the figure 1 that, for a given initial of sea surface and seabed, the profile of the bottom has modified under the influence of soliton. We observed regions that have suffered erosion and others an agglomeration. In addition, we can observe that the configuration of the profile bottom affects the amplitude of wav propagation. These results showed that the morphodynamic of seabed depend on the importance of interaction of fluid and sediment.

1.6. Effect of water-sediment interaction parameter

The figures (figure 2) below illustrate the morphodynamic of seabed on the shallow water for four different cases of values of interaction parameter Ag at fixed time t = 45 s


Elevation (m)
x - coordinate (m)
Zs, Ag = 0.05
Zb, Ag = 0.05
Elevation (m)
x - coordinate (m)
Zs, Ag = 0.1
Zb, Ag = 0.1
Elevation (m)
x - coordinate (m)
Zs, Ag = 0.15
Zb, Ag = 0.15

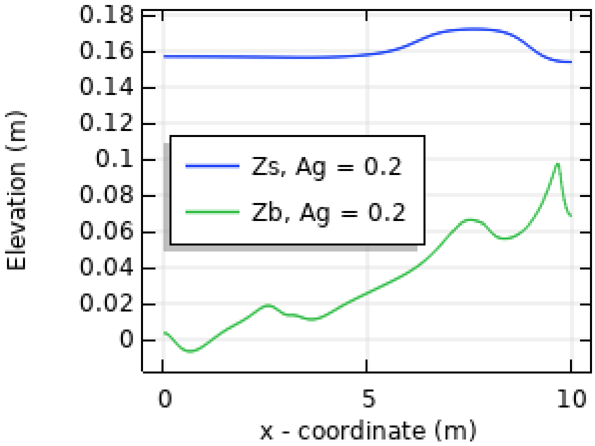


Figure 2. Morphodynamic of seabed with Ag = 0.05, Ag = 0.1, Ag = 0.15 and Ag = 0.2

these figures, we can observe that the variation of interaction parameter significantly affects the profile of seabed. Indeed, the zones of erosion and sediment deposition are increasingly important when the value of the interaction parameter Ag increases. This is explained by the mode of transport of the sediment, its particle size and the viscosity of the fluid. We can deduce that que the variation of seabed depend of the characteristics of the seabed and the hydrodynamic of the water. This affirmation is verified on the literatures (confer [10], [11]).

IV. Conclusions

To conclude, the anticipation of the variation of seabed is important, particularly, for many people who live in the coastal zone. In this work, we have attached ourselves to studying the evolution of seabed. So we propose a numerical model capable of simulating the spatial-temporal evolution of the floor topography. Our model consists of Saint – Venant system for the part of hydrodynamic coupled with the equation Exner for the part transport of sediment. We discretize and solve the PDE system by the finite element method on the COMSOL Multiphysics. To verify our model, we simulate the action of propagation wave on the mobile floor and we compare our results with the literatures. The results that we obtained, are the variation over time of the level of sea surface and the seabed under the action of propagate wave. In addition, we took a range value of Ag parameter and we analyze their effects on the topography of bottom. These results showed that the wave behavior affect the profile of seabed and vice versa the shape of seabed modify the amplitude of wave. The analyze of interaction parameter Ag allowed us to conclude that the increase the value of Ag reinforces the areas erosion and agglomeration. That implies that profile of topography evolution depend on the characteristic of seabed and the hydrodynamic. These results are in agreement with literatures.

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