



Using Some Two Parameters Distributions for Modelling Daily Exchange Rate Singapore Dollar to Indonesia Rupiah Data

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ABSTRACT: One of the nations that affects Indonesia's economic growth is Singapore, a developed nation in Southeast Asia. Thus, it is crucial to use probability modelling to examine the change in currency exchange rates from the Singapore dollar (SGD) to the Indonesian rupiah (IDR). This work focuses on predicting daily exchange rate movements of SGD to IDR utilizing a variety of two-parameter probability models, including Weibull, Gamma, Log Normal, and Normal. using graphical techniques like density plots and cumulative plots as well as numerical techniques like Akaike information criterion (AIC) and Bayesian information criterion to determine the optimum model or The Goodness of Fit (GOF) (BIC). Following that, the GOF between theoretical data and model distributions is assessed.

Keywords: Exchange rate, GOF, Wiebull, Gamma, log normal, normal distribution.

I. INTRODUCTION

The value of one currency in relation to another is determined by the exchange rate between the two currencies. One of the most crucial elements for economic progress in many nations is a fair or reasonable exchange rate (Sarpong, S., 2019). According to Aron et al., exchange rates have a direct impact on a nation's external credit, employment, trade flows, balance of payments, and production and consumption agreements (1998). Future exchange rate estimates must take into account both value and probability. To forecast exchange rates, some scholars turn to statistical theory. The Box-Jenkins approach of time series modeling and forecasting is popular among researchers. The Box-Jenkins technique for forecasting has significant drawbacks. This approach presupposes that the variables have a linear connection to one another. But in the real world, time series data are usually non-linear, Lin et al. (2012), Huang et al. (2010), Dinger et al. (2009) and Gradojevic and Yang (2006). (2006). Second, Ridhwan et al. (2015) found that the researcher's skill and experience are highly correlated with the model selection procedure when using the Box-Jenkins method. As a result, the accuracy of modeling and forecasting using the Box-Jenkins approach is insufficient. The paper instead investigates the characteristics of probabilistic modeling before presenting data. The fact that probabilistic modeling is grounded in actual data makes it predictive. Additionally, they are able to reflect immediate changes in the data and do not require lengthy historical time series for effective estimation.

Moreover, the inherent uncertainty in the data can be effectively captured via probabilistic modeling. The mathematical foundation established by Chang and Melick has comparatively little impact on probabilistic modeling (1999). In most cases, probabilistic models choose the probability density function that fits the data's frequency the best. Researchers from a variety of disciplines have employed probabilistic modeling extensively. Examples include probabilistic models for diabetic patients and models to predict chronic diseases brought on by the Covid-19 virus (Hendri Fahrizal et al., 2023; Manda Lisa Usvita, 2021). For example, short-term rainfall probability modeling (Muhammad Rajab et al., 2022) (Halimah Tun Satdiah et al., 2023) is a

climatic variable that benefits from probabilistic modeling (Lisa Rahayu et al., 2023). Probability models are frequently used to explain temperature climatic variables to describe the frequency of temperatures in an area (Martha Sri Pramadhani et al, 2021) If wind speed frequency data is analyzed using a probability model, wind speed as a climatic variable that is valuable as a renewable energy source will also be highly useful (Arian Syaputra. et al, 2022). It is difficult to model exchange rate probabilities, and this field of study is still highly active. Many attempts to create probability models from diverse perspectives have been examined over the past ten years. Researchers are still looking for a theoretical probability distribution model that matches the empirical distribution of spot exchange rate fluctuations. Improved test statistics and more accurate pricing models for exchange rates should result from improved theoretical models of these empirical distributions. For many years, scholars believed that a normal or lognormal probability distribution best captured the empirical distribution of changes in exchange rates. Likewise, a number of recent studies and tests "automatically" translate returns from changes in spot, forward, and futures exchange rates using logarithms; this transformation presumes, expressly or implicitly, that the transformed data yields returns that are normally distributed (Johnston, K & Scoot, E, 1999).

To explain variations in exchange rates, four probability distributions have been put forth: normal, stable paretian symmetric, student's t, and mixed normal. The findings indicate that the log of exchange rate changes for daily data has an unusual distribution. The best match is provided by the Student distribution and a combination of two normal distributions (Boothe and Glassman, 1987). Those that deal in currency exchange only have one goal in mind: to minimize loss due to fluctuations in the exchange rates between the currencies. So, being able to predict the exchange rate precisely maximizes the profits. As a result, our study has sufficiently demonstrated that the Ghana Cedi to US Dollar exchange rate follows the lognormal distribution. The maximum returns in profit will be obtained by using the Lognormal distribution to anticipate the potential exchange rate between the two currencies (Sarpong, S., 2019). This study takes into account the skewed t, skewed generalized error, generalized t, and skewed generalized t probability distributions. The parameters of each model are estimated using the greatest likelihood method. The Akaike information criterion (AIC) and Bayesian information criteria are used to compare and choose models (BIC). The study's findings indicate that the daily BWP/USD exchange rate series is out of the ordinary and adversely skewed with heavy tails.

The maximum returns in profit will be obtained by using the Lognormal distribution to anticipate the potential exchange rate between the two currencies (Sarpong, S., 2019). This study takes into account the skewed t, skewed generalized error, generalized t, and skewed generalized t probability distributions. The parameters of each model are estimated using the greatest likelihood method. The Akaike information criterion (AIC) and Bayesian information criteria are used to compare and choose models (BIC). The study's findings indicate that the daily BWP/USD exchange rate series is out of the ordinary and adversely skewed with heavy tails. Maximum likelihood parameter estimation and model selection using graphical techniques like density plots and cumulative plots as well as numerical techniques like Akaike information criterion (AIC) and Bayesian information criterion (BIC)

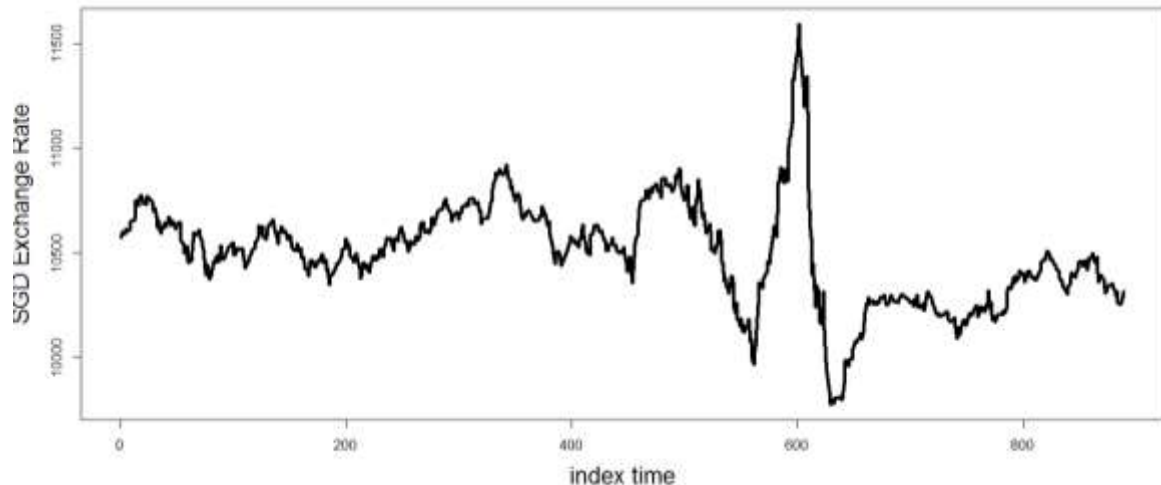


Fig. 1. Daily Exchange Rate SGD/IDR Data

II. MATERIAL AND METHODS

2.1. Data

The Singapore Dollar to Indonesian Rupiah exchange rate (SGD/IDR) is included in the data set. These numbers were taken from the website of the Central Bank of Indonesia (BI) (www.bi.go.id). There were 889 observations in the data collection, which covered the time period from February 1, 2019, to September 9, 2022. Figure 1 clearly demonstrates the daily SGD/IDR exchange rate. In Table 1, some condensed statistics are provided. Table 1 provides introductory information regarding the SGD/IDR exchange rate in the form of statistical data. The values displayed are descriptive statistics, such as mean, variance, kurtosis, skewness, maximum and minimum data, that usually describe the data obtained. Table 1 shows that the skewness value is quite low and the kurtosi value is below 1, which may indicate the use of unbalanced probability models like Weibull and Gamma. In this study, exchange rate data modeling options include log normal and normal. This is also made clear by the data histogram in Figure 2, which demonstrates that the four probability models employed in this investigation were chosen for the proper reasons.

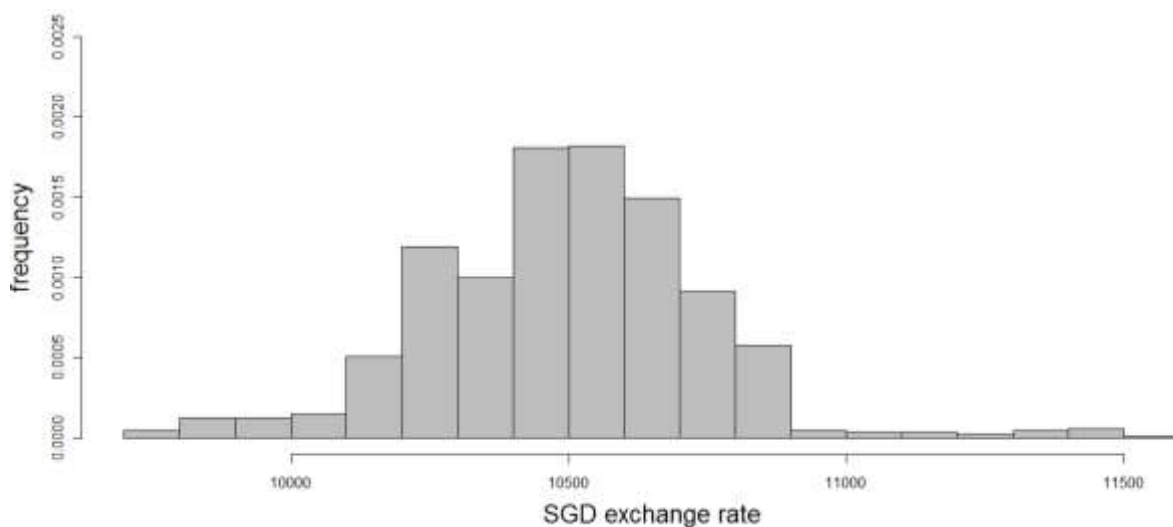


Fig. 2. The Histogram Daily Exchange Rate SGD/IDR

mean	varians	minimum	maximum	skewness	kurtosis
10498.12	60225.34	9777.73	11592.53	7.18×10^{-12}	0.14

Table 1: Statistics of daily Exchange Rate SGD/IDR.

2.2 Probability Density Function

The following is a description of four daily exchange rate models along with their associated probability density functions. X , the random variable corresponding to the daily exchange rate, should be noted. The shape and scale parameters are denoted by the two parameters of the weibull distribution, respectively.

$$f(x) = \frac{\theta}{\lambda} \left(\frac{x}{\lambda}\right)^{\theta-1} \exp\left[-\left(\frac{x}{\lambda}\right)^{\theta}\right], \theta > 0, \lambda > 0, x > 0$$

The gamma distribution with two parameters, α and β denote the shape and scale parameters respectively.

$$f(x) = \frac{x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right)}{\beta^{\alpha} \Gamma(\alpha)}, \alpha > 0, \beta > 0, x > 0$$

The scale parameter sets the variation of the exchange rate series, which is given in the same unit as the random variable X , and the shape parameter controls the shape of the exchange rate distribution. The gamma distribution has the same characteristics of nonnegativity and positive skewness as the weibull distribution. The location and scale parameters are indicated by and, respectively, in the Log Normal with two parameters. Meanwhile, X is the daily SGD/IDR exchange rate. The following is the probability density function:

$$f(x) = \frac{1}{x\sqrt{2\pi\eta^2}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \kappa}{\eta}\right)^2\right), \kappa > 0, \eta > 0, x > 0$$

A continuous random variable X is normally distributed or follows a normal probability distribution if its probability distribution is given by the following function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(x - \mu)^2}{\sigma^2}\right), -\infty < \mu < \infty, 0 < \sigma^2 < \infty, -\infty < x < \infty$$

with mean μ and variance σ^2 .

2.3 Maximum Likelihood

There are numerous techniques for estimating parameters, including the least squares method (LS), generalized probability weighted moments, the method of moments (MM), maximal likelihood estimation (MLE), and L-moments (GPWM). The MLE approach is taken into consideration in this study because, when compared to other methods, it offers the least variance. Finding a set of parameters that will optimize the likelihood function is the goal of this approach. Differentiating the log likelihood function with respect to the distribution's parameters yields the parameters. We won't go into detail in this work about the implicit form and complexity of the maximum likelihood equation for this distribution. To solve the equation, however, the Newton-Raphson method was used in an iterative process. For this process, various beginning values have been tested. The initial value is thought to be the selected estimated parameter if it tends to converge to similar values and has the highest likelihood.

2.4 Goodness-of-fit Tests (GOF)

Results from multiple goodness-of-fit tests are used to determine which distribution is the best appropriate (GOF). The GOF tests taken into consideration are based on numerical criteria, graphical inspection probability density function (pdf), and cumulative density function (cdf). To ascertain the distributions' goodness-of-fit standards, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used. Although graphical inspection typically produced the same result, AIC, BIC, and Log

Likelihood ($\ln(L)$) results varied. The distribution with the lowest AIC and BIC values was determined to be the best fit outcome. The following is the formula for calculating AIC and BIC:

$$AIC = -2 \ln L + 2k, \quad BIC = -2 \ln L + k \ln n,$$

where k = the number of parameters, n = the sample size

III. RESULT

The MLE approach will be used in this study to estimate the parameters of the four probability distributions, as shown in table 2. These parameters will be used to generate pdf plots for all distributions in this study, which will be used to verify the model's goodness of fit by comparing how closely the plots resemble the daily exchange rate SGD/IDR histogram data shown in Figure 3. This figure illustrates how the models employed have several ways of addressing the histogram. Figure 3 demonstrates that the histogram or frequency of Exchange Rate SGD/IDR data can be approached by two parameter distributions, such as Gamma Log Normal and Normal.

Parameter	Weibull	Gamma	Log Normal	Normal
θ	36.95	-	-	-
λ	10620.25	-	-	-
α	-	1920.00	-	-
β	-	0.18	-	-
κ	-	-	9.25	-
η	-	-	0.023	-
μ	-	-	-	10498.11
σ	-	-	-	245.27

Table 2: Estimated parameters for probability density function.

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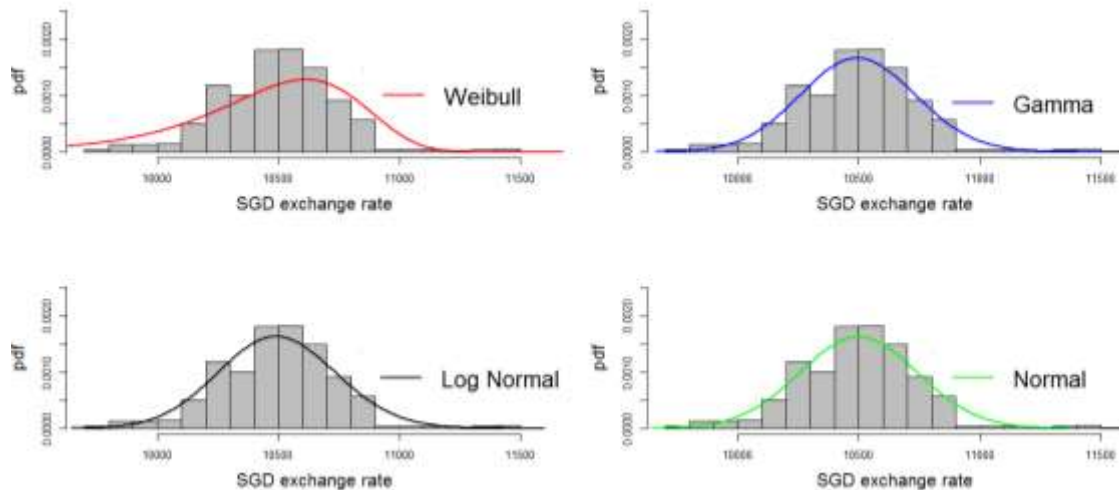


Fig. 3. A plot of the histogram and theoretical Weibull densities (red line), Gamma densities (blue line), Log Normal (black line) and Normal (green line) for the exchange rate between the Singapore Dolar (SGD) and Indonesia Rupiah (IDR)

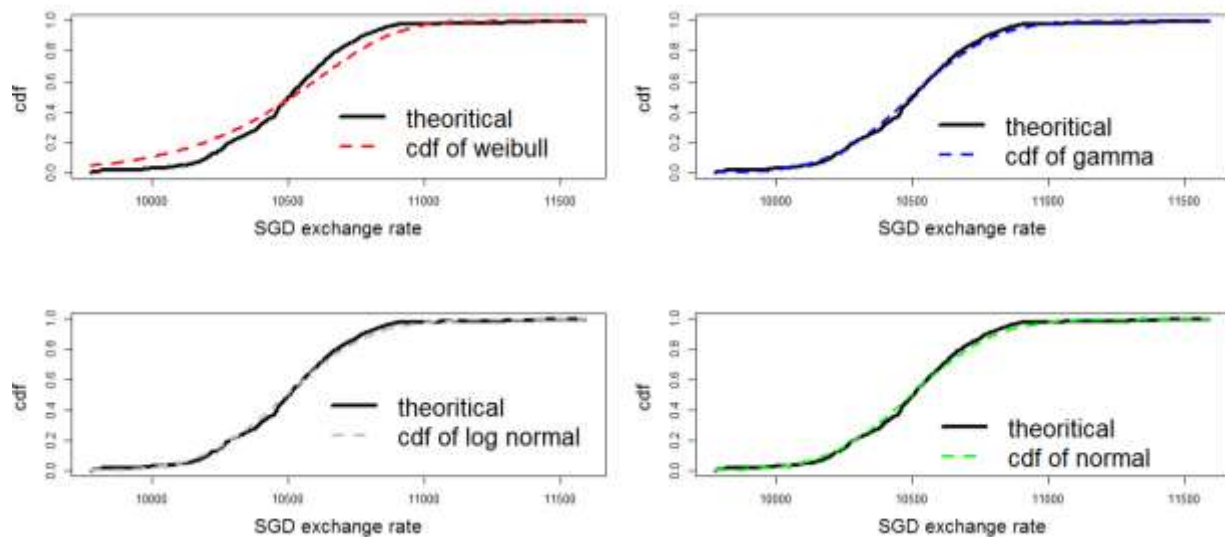


Figure 4. Empirical and theoretical cumulative distributions functions (cdf) of Weibull (red line), cdf of gamma (blue line), cdf of log normal (grey line) and cdf of normal (green line) for the exchange rate between the Singapore Dolar (SGD) and Indonesia Rupiah (IDR).

IV. CONCLUSION

The probability of the daily exchange rate SGD/IDR occurring was looked at in this paper. To fit the data, the four probability distributions of Gamma, Weibull, Log-Normal, and distribution were chosen. MLE was used to specifically estimate the parameters of the four probability distributions by analyzing the various types of data in this work. It could be demonstrated in this paper that the MLE was effective in this study. This study focuses on analyzing the frequency of daily SGD/IDR exchange rate data to determine the appropriate models or distributions for describing the data's distribution.

Probability Models	AIC	BIC	Log Likelihood
Weibull	12603.82	12613.40	- 6299.91
Gamma	12308.00	12317.58	- 6152.00
Log Normal	12306.04	12315.62	- 6151.02
Normal	12310.07	12319.65	- 6153.03

Table 3. The Goodness-of-fit Criteria for the exchange rate between the SGD/IDR.

When compared to other well-known distributions, the Log Normal two parameter distribution produced superior results. This conclusion is based on well-known goodness of fit test models like AIC, BIC, and Log L. Additionally, graphical techniques like pdf and cdf plots were used to compare the empirical distributions with the adjusted Log Normal two parameter distribution.

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