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The Comparison of Generated Synthetic Monthly Rainfall Using Some Two Parameters Distribution

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ABSTRACT: The ability to generate synthetic monthly rainfall characteristics is important for predicting hydrological impacts resulting from land use and climate change in the humid tropics. We present some quantile functions from two parameter distributions such as gamma, weibull and log normal to generate synthetic monthly rainfall data, which we demonstrate for the city of Pekanbaru, Riau Province of Indonesia. We use maximum likelihood to estimates the parameters of the probability density function. Synthetic rainfall will be generated 100 times using different quantile functions. Every time Synthetic Rainfall is generated, two statistical such as mean and maximum monthly rainfall will be produced and displayed in graphical form. The same statistics are also produce from the historical rainfall data, the ability of the generated synthetic rainfall graphs to capture the historical rainfall graph will show the ability of the probability density function to generate monthly synthetic rainfall. In this study, Overall, the statistics produced by synthetic rainfall using the gamma distribution can closely approximate the statistics produced by historical rainfall data when compared to the synthetic rainfall produced by the other two distributions.

Keywords: Synthetics Rainfall, Quantile Function, Gamma Distribution, Weibulll Distribution, Log Normal Distribution.

I. INTRODUCTION

The realistic to generate of continuous synthetics rainfall series is required for several purposes in hydrology. Many large-scale ecological and water resource models require daily, monthly and yearly rainfall data as input to the model. As historical data provide only one realisation, stochastically generated rainfall totals are necessary to assess the impact of rainfall variability on water resource systems, including hydrological risk. Hydrological risks can be approached using models to generate synthetics rainfall. On the other hand, generated synthetics rainfall is a necessity for water resources management, drought, groundwater studies, floods, runoff, food security, and other hydrological, hydraulic, and even agricultural issues [1].Synthetic rainfall data provide alternative realisations that are equally likely but have not necessarily occurred in the past [2]. As a result, the modelling of rainfall records has become well established over the past 30 years. Studies on generated synthetics rainfall behavior have attracted much attention from scientists throughout the world, such as In particular,Previously, the linear regression method was used to generated synthetics rainfall and time series. In 1987, Lapedes and Farber [3] showed that intelligent methods provided better efficiency in generated synthetics rainfall time series. Currently, various intelligent methods are used to generated synthetics rainfall most of which have a better ability to solve non-linear hydrological phenomena compared to regression and statistical methods [4,5,6,7,8,9,10,11]. Valverde Ramírez et al. [12]

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Investigated the application of artificial neural network method in the generated synthetics rainfall in Sao Paulo, Brazil. The results highlighted the higher accuracy of the predictions made by this method than that of other methods. Using artificial neural networks and comparing it with multivariate regression, Dahamsheh and Aksoy [13] generated synthetics monthly rainfall in Jordan's arid regions and revealed the better performance of the artificial neural network compared to other methods. Huo et al. [14] used hydrological and agricultural data in an integrated ANN model and compared its performance with the Lumped artificial neural network and the linear regression model. The results of this research showed the better performance of the integrated ANN model in estimating the monthly flow at the output. This models assumes that there is a linear relationship between the variables. But in the real world, time series data are usually non-linear, Lin et al. (2012)[15], Huang et al. (2010)[16], Ding et al. (2009)[17] and Gradojevic and Yang (2006)[18]. Second, the method is highly dependent on the ability and experience of the researcher, Radhwan et al., 2015[19]. Therefore, modeling and forecasting using this method does not provide sufficient accuracy. Instead, the paper examines the nature of probabilistic modeling, followed by data. Probabilistic modeling is predictive because it is based on real data. Furthermore, they do not require long historical time series for accurate estimation and are also able to reflect instantaneous changes in the data. Additionally, probabilistic modeling is well suited to capturing the inherent uncertainty in the data. Probabilistic modeling is relatively unaffected by the mathematical precedent of Chang and Melick (1999)[20]. Probabilistic models typically determine the best-fit probability density function with respect to the frequency of the data. Probabilistic modeling has been widely used by researchers in various fields, including probabilistic models for determining chronic diseases caused by the Covid-19 virus (Hendri Fahrizal et al., 2023)[21], probabilistic models for diabetic patients (Manda Lisa Usvita, 2021)[22] .

Probabilistic modeling is also well suited for climate variables, such as short-term rainfall probability modeling (Muhammad Rajab et al., 2022)[23] (Halimah Tun Satdiah et al., 2023)[24] (Lisa Rahayu et al., 2023)[25]. Temperature climate variables also often use probability models to describe the frequency of temperatures in an area (Martha Sri Pramadhani.et al, 2021)[26] wind speed as a climate variable that is useful as a renewable energy source will also be very useful if wind speed frequency data is modeled with an probability model (Arian Syaputra. et al, 2022)[27]. The probability model is also very useful in generating synthetic monthly rainfall. In particular, the gamma distribution has been used many times to model rainfall totals on wet days. Valuable general reviews on weather generators are published by [28] and [29]. More elaborate models have been proposed for the distribution of precipitation amounts given the occurrence of a wet day. Stern and Coe (1984)[30] used the two-parameter gamma distribution to describe the precipitation amount on wet days. An excellent review of stochastic weather models has been presented [31]. Although a large number of precipitation models have been developed, many practical applications require that weather generators produce other meteorological variables in addition to precipitation. Synthetic rain data generation for various time periods such as daily, monthly and yearly is well done in Australia, the quantile function of the two-parameter gamma probability density function plays a very important role in this purpose [32]. Several algorithms have been developed for the purpose of generating daily, monthly and yearly synthetic rainfall. Rainfall generation algorithm (rGen) has been generated to produce annual synthetic rainfall [33] and new synthetic daily rainfall generated together [34] with a similar model to produce monthly synthetics total.This study focuses on using several two-parameter probability models such as Weibull, Gamma and Log Normal in generating synthetics monthly rainfall based on 100 time simulation using the quantile function of probability models. Parameter estimation using the maximum likelihood method and selecting the best probability model using graphical technique Especially in the ability of each generated synthetic rainfall resulted by each probability models in capturing the mean and maximum value of rainfall observation every month

II. DATA AND STUDY AREA

Pekanbaru is a big city which is the capital of Riau Province. Pekanbaru which has a tropical climate, with mothly rainfall with varying amount. The original data consisted of daily rainfall records from 1990 to 2008, which were provided by the Meteorological, Climatological, and Geophysical Agency of Pekanbaru, Indonesia. The initial information about the nature of mothly rainfall in Pekanbaru can be seen in table 1. Some basic statistics are displayed based on daily rainfall data in Pekanbaru. From table 1 it can be seen that the month with most precipitation is April with precipitation total 365.82 milimeters, while the least precipitation is June with precipitation total 143.91 milimeters. From table 1 it also can be seen that the skewness value is very small this can mean that unequal probability models such as Weibull, Gamma and Log Normal can be used in modeling monthly rainfall data in this research.

	mean	varians	minimum	maximum	skewness	kurtosis
January	274.36	28679.3	73	774.4	3.00×10^{-7}	7.15
February	184.96	12934.49	53.5	540.9	1.01×10^{-6}	7.98
March	358.27	122969.3	108.1	1684.6	6.63×10^{-8}	15.87
April	365.82	89776.91	102.8	1405.6	8.25×10^{-8}	11.61
May	304.18	141666	105	1797.4	6.21×10^{-8}	19.15
June	143.91	5963.55	51.5	270.3	7.12×10^{-7}	2.12
July	187.35	16977.6	12.2	476.4	3.84×10^{-7}	3.91
August	160.61	7665.427	51.2	432.7	2.21×10^{-6}	7.72
September	202.48	12954.41	53.8	451.4	2.29×10^{-7}	3.07
October	287.77	17288.66	140	622	4.36×10^{-7}	4.59
November	313.80	9915.467	168.7	480.5	-1.38×10^{-7}	2.56
December	339.82	49040.72	125.5	1100.8	1.89×10^{-7}	11.07

Table 1 Statistics of monthly rainfall observation

III. METHODOLOGY

1.1 Probability Density Function

Three probability models are described as follows with their probability density functions. Note that X is the random variable representing the daily exchange rate. The weibull distribution with two parameters, θ and λ denote the shape and scale parameters respectively.

$$f(x) = \frac{\theta}{\lambda}\left(\frac{x}{\lambda}\right)^{\theta-1} exp\left[-\left(\frac{x}{\lambda}\right)^{\theta}\right], \theta > 0, \lambda > 0, x > 0$$

The gamma distribution with two parameters, α and β denote the shape and scale parameters respectively.

$$f(x) = \frac{x^{\alpha-1}\exp\left(\frac{-x}{\beta}\right)}{\beta^{\alpha}\Gamma(\alpha)}, \alpha > 0, \beta > 0, x > 0$$

The shape parameter governs the shape of the exchange rate distribution and the scale parameter determines the variation of exchange rate series which is given in the same unit as the random variable X. Slightly similar to the weibull distribution, the gamma distribution shares the same properties of nonnegativity and positive skewness. The Log Normal with two parameters, κ and η denote the location and scale parameters, respectively. The probability density function is given as follows

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right), \mu > 0, \sigma > 0, x > 0$$

1.2 Maximum Likelihood (MLE)

Many methods are available for parameter estimations, which include the method of moments (MM), maximum likelihood estimation (MLE), the least squares method (LS), L-moments and generalized probability weighted moments (GPWM). The MLE method is considered in this study because it provides the smallest variance as compared to other methods. The idea of this method is to find a set of parameters that will maximize the likelihood function.The parameters are obtained by differentiating the log likelihood function with respect to the parameters of the distribution. The maximum likelihood equation for this distribution is in implicit form and complicated and we will not discuss details in this paper. However, Newton-Rhapson's method has been employed in iteration procedure to solve the equation. Different initial values have been tested for this procedure. If the initial value converge to same vales and have the largest likelihood, it is considered to be the choosen estimated parameter.

1.3 Generated Synthetic Monthly Rainfall Using Gamma Quantile Function

A large number of synthetic monthly totals are generated using a two-parameter gamma distribution. The two parameters, θ and λ, used to describe the weibull distribution, α and β used to describe gamma distribution whereas μ and σ used to describe log normal distribution are found using maximum likelihood estimation. To generate a sequence $\{x[t]\}$ of synthetic monthly rainfall, we first generate realisations $\{r[t]\}$ of a sequence $\{R[t]\}$ of independent random numbers, each one uniformly distributed on the unit interval [0, 1], and then use quantile functions to solve the equation

$$F^{-1}[\alpha[t], \beta[t]](x) = r[t]$$

to find the corresponding monthly rainfall denoted by $x = x[t]$. $\alpha[t]$ and $\beta[t]$ are defined by the maximum likelihood estimates from the observed monthly data. using the same method can be applied to the weibull and log normal quantile function for the purpose of generating monthly synthetic rainfall, preceded of course by parameter estimation for each probability density function.

IV. RESULT

The parameters of the probability density function distributions such as Gamma, Weibull and Log Normal distribution are estimated using the maximum likelihood estimation method. Table 2 presents the estimated parameters of all distribution based on the historical monthly rainfall data. Based on the parameters shown in table 2, synthetic rainfall will be generated every month for 100 times of generation. The quantile function or invers of the distribution function will be used for this purpose. Some statistics such as mean and maximum value every month for every 100 times the generate synthetic rainfall will be compared with the same statistics generated from the historical data. Both statistics generated through synthetic rainfall and historical data will be displayed using graphs. The ability of the statistics from synthetic rainfall graph to capture the historical rainfall graph is a marker that the probability density function can be used properly in generated synthetic rainfall.

	Weibull		Gamma		Log Normal	
	θ	λ	α	β	μ	σ
January	1.795	310.41	2.62	104.53	5.45	0.589
February	1.799	209.20	2.64	69.93	5.06	0.593
March	1.318	395.16	1.04	343.22	5.64	0.626
April	1.445	407.91	1.49	245.42	5.67	0.661
May	1.158	324.94	0.65	465.72	5.41	0.662
June	2.060	163.35	3.47	41.43	4.82	0.577
July	1.496	207.36	2.07	90.62	4.95	0.888
August	2.015	182.16	3.36	47.72	4.96	0.501
September	1.919	228.77	3.16	63.97	5.13	0.664
October	2.392	325.83	4.79	60.08	5.57	0.431
November	3.726	348.71	9.93	31.59	5.69	0.351

December	1.739	384.68	2.35	144.31	5.67	0.543

Table 2 Estimated Parameters for Some Two Parameters Distributions

The mean generate synthetic monthly rainfall summarized using 100 black lines can very well capture the mean of monthly rainfall from the actual data represented by the red lines for each month. This situation explains the conclusion from Figure 1, that it is clear that the red line is within 100 black lines. This also explains that the black line is the mean generate synthetics monthly rainfall using the gamma distribution which almost the same as mean of historical rainfall data.

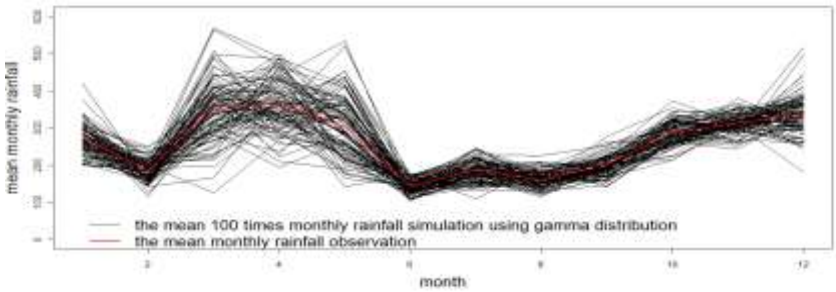

Figure 1 Mean Monthly Rainfall based on 100 time simulation (black lines) using gamma distribution vs observarion (red line)

The results are quite significantly different as shown in Figure 2. The figure clearly shows that the Weibull distribution is not good to generate synthetics monthly rainfall, the mean of synthetics rainfall through 100 simulations represented by the blue line cannot capture the mean monthly rainfall the historical data represented by the red line, especially in February, June, July, August and September. Therefore it can be concluded that the Weibull probability density function is not good enough to be used to generate synthetic monthly rainfall. The synthetics rainfall is also generated using the normal log distribution. 100 simulations using the log normal distribution quantile function were carried out to produce monthly rainfall which is represented by the green line. The ability of this synthetic rainfall to capture monthly rain from historical data which is represented by the red line is also shown in Figure 3. Figure 3 shows that the probability density function of log normal distribution is good enough to generated synthetics rainfall for every month, it is shown that the red line is well captured by a collection of lines green. From the three probability density functions used, it can be concluded that the gamma and log normal distributions can be used to generate synthetic monthly rainfall which has good quality marked by the ability of generate synthetics monthly ranfaill that almost resembles the mean of historical monthly rainfall data.

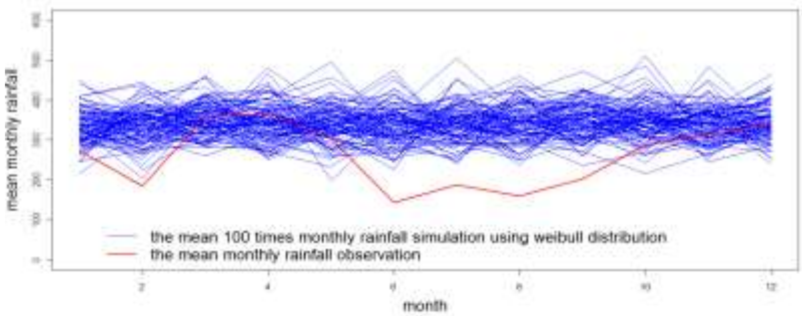

Figure 2 Mean Monthly Rainfall based on 100 time simulation (blue lines) using weibull distribution vs observarion (red line)

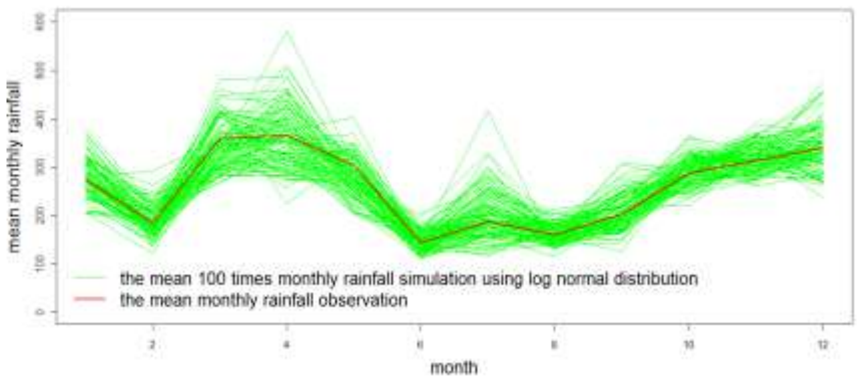

Figure 3 Mean Monthly Rainfall based on 100 time simulation (green lines) using log normal distribution vs observarion (red line)

Furthermore, to clarify the ability of the probability density function to generate synthetics rainfall, the mean value of 100 simulations will be represented by an one value by taking the average of the 100 simulated values. Figure 4 shows a barplot of the mean observed monthly vs the mean of 100 times generated monthly rainfall using the two parameter distribution such as gamma, weibull and log normal for January to December. In this figure the blue, black, red and green bars represent the mean of historical data, the mean rainfall from 100 simulations using the gamma, the Weibull and the log normal distribution respectively. Almost all months in this study the gamma distribution can generate synthetic rainfall which is capable of producing mean monthly rainfall which is almost the same as the historical data, as shown by the blue bars which are almost the same as the black bars.

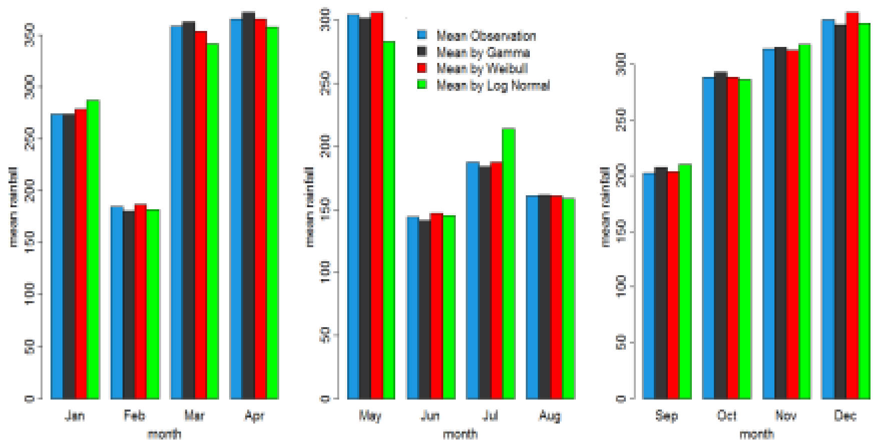

Figure 4 Mean Monthly Rainfall based on average of 100 time simulation vs observarion

Statistics in the form of the maximum value of generated synthetics monthly rainfall using different probability density functions are also analyzed to clarify the results of this study. From the figure 5 can be seen that The maximum generate synthetic monthly rainfall represent by using 100 grey lines can capture the maximum of monthly rainfall from the historical data represented by the red lines for each month. This also explains that the black line is the maximum generate synthetics monthly rainfall using the gamma distribution which almost the same as maximum of historical rainfall data. Figures 6 and 7 represent of the 100 maximum synthetics monthly rainfall using the Weibull distribution represented by the orange line and the normal log distribution represented by the yellow line respectively. From Figure 6 it can be seen that 100 orange lines cannot capture the red line well in certain months, such as March, April, May, June, July, August, September, October and November. This can be interpreted that some of the maximum rainfall values from the historical data cannot be captured by the maximum the synthetics monthly rainfall using the Weibull distribution. From Figure 7 it can also be seen that using the probability density function of log normal distribution to generate synthetics monthly rainfall gives good results in approaching the maximum monthly rainfall from the historical data. This can be seen from the 100 maximum values of synthetic which are represented by the yellow line and can capture the maximum value of rainfall from the historical data which is represented by the red line.

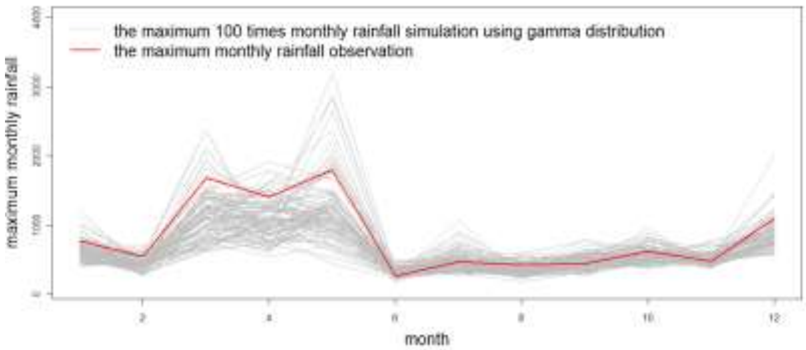

Figure 5 Maximum Monthly Rainfall based on 100 time simulation (grey lines) using gamma distribution vs observarion (red line)

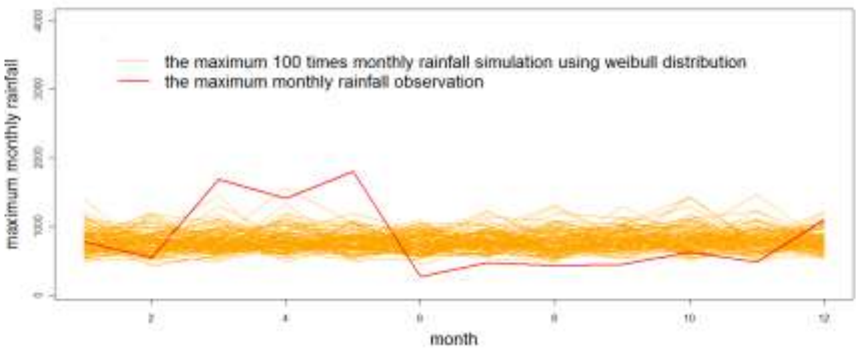

Figure 6 Maximum Monthly Rainfall based on 100 time simulation (orange lines) using weibull distribution vs observarion (red line)

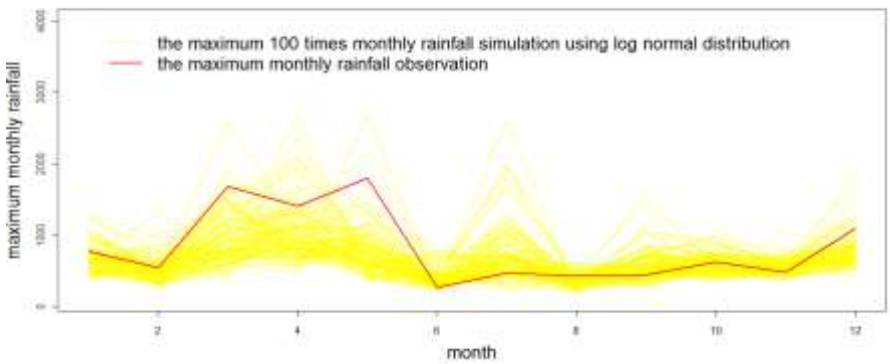

Figure 7 Maximum Monthly Rainfall based on 100 time simulation (yellow lines) using log normal distribution vs observarion (red line)

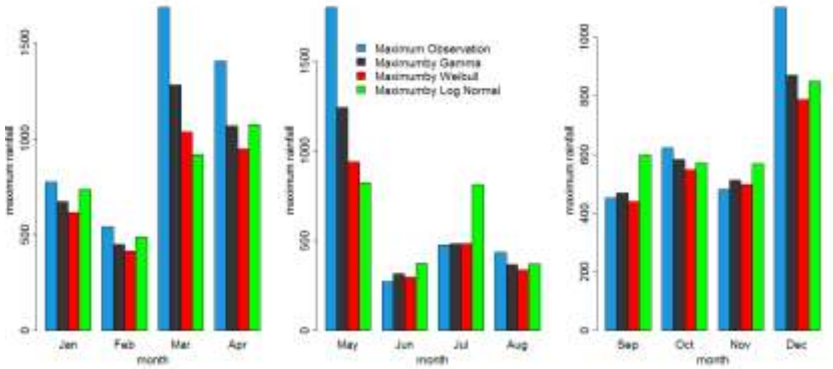

Figure 8 Maximum Monthly Rainfall based on average of 100 time simulation vs observarion

Figure 8 shows a barplot of the maximum observed monthly vs the maximum of 100 times generated monthly rainfall using the two parameter distribution such as gamma, weibull and log normal for January to December. In this figure the blue, black, red and green bars represent the maximum historical data monthly, the maximum rainfall from 100 simulations using the gamma, the Weibull distribution and the normal log distribution respectively. Almost the same results are shown by figure 8, the ability of synthetics rainfall to approach the monthly maximum rainfall for historical data. Most of the months show that the gamma distribution is very good for generate synthetics rinfall, although for certain months such as March, April, May and December the gamma distribution is not very good at generate synthetics rainfall which has a maximum rainfall that differs quite significantly from the historical rainfall data. Overall it can be concluded that the gamma distribution is the best used to generate synthetics rainfall compared to the other two distributions.

V. CONCLUSION

In this paper, synthetic monthly rainfall is generated using the qantile function of the distribution of two parameters, namely Gamma, Weibull and Log. The synthetic rainfall will be generated 100 times. Two statistics such as mean and maximum rainfall will be generated from the generated synthetic rainfall and will be compared with the same statistics from the historical rainfall data. Synthetic rain generated from different probability density functions is said to be the best if it can capture statistics form historical data. Overall the synthetic rain produced using the probability density function of gamma is the best of the other synthetic rainfall.

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